

Ride Comfort of a 4 DOF Non Linear Heavy Vehicle Suspension

Md. Zahid Hossain¹ and

Md. Nurul Absar Chowdhury²

Islamic University of Technology (IUT)

Board Bazar, Gazipur-1704, Bangladesh

Email¹: zahidmce@iut-dhaka.edu

Email²: nabsar@iut-dhaka.edu

Abstract

Vibration of a vehicle is directly influenced by road roughness and suspension system. Apart from the road roughness, proper vehicle suspension system can play vital role to provide the driver and passenger comfort. In this literature, a non linear mathematical model of 4 degree of freedom (DOF) heavy vehicle suspension is derived with half car model. Vibration characteristics due to parametric changes such as pitch line excitation and damping coefficients are

investigated for heavy vehicle suspension system. The model is simulated by MATLAB SIMULINK to observe the vibration phenomena. Ride comfort is investigated by amplitude of vibration and ride comfort level. A comparison between nonlinear and linear spring model has also been investigated briefly.

Keywords: 4DOF, Half car model, MATLAB SIMULINK, Nonlinear spring, Ride comfort.

1. Introduction

Travelling on a rough road surface especially observed in off road surface and road surfaces in developing countries mainly prompts the vehicle vibration which influences the longevity of suspension system and the rider comfort. Investigation on car suspension system can help to recommend the permissibility of range of road roughness and choose the proper suspension system which can withstand the vibration as well as provide the rider comfort.

A MATLAB and SIMULINK based project work on vibration of a quarter car 2 DOF model and demonstrated the significance of using this software [1]. Mostly quarter car [1-5] linear passive suspension models have been observed in the literatures for investigations of bounce and pitch motions of the vehicle body. Parametric sensitivity of a heavy duty vehicle suspension system has been demonstrated in a literature[3]. The authors demonstrate the rider comfort on the basis of vehicle body acceleration and dynamic tire loads for the quarter car model with only the bounce motion for vehicle body and tire.

Demonstrations for active or semi active suspension system of MDOF models have been found in some literatures [4,6]. These literatures aim to reduce the vibration of a vehicle body using the complicated mechanisms such as actuators, hydraulic systems, feedback control, etc.

The main objective of this paper is to study the heavy vehicle body bounce and pitch motion for half car nonlinear model considering 4 DOF system. Damping coefficients and road roughness parameters have been changed to check the rider and passenger comfort.

2. Modeling of vehicle suspension system

A physical model of half car model for heavy vehicle suspension system is shown in **Figure 1**. Sprung mass, unsprung masses, and associated springs and dampers for the front and rear parts are presented in this figure. As the heavy leaf springs in the suspension system of heavy vehicle and the heavy tires in the wheels are used, it is practical to consider as nonlinear spring. An analytical model of this suspension system is shown in **Figure 2**. x_s and θ_s represent the bounce and pitch motion

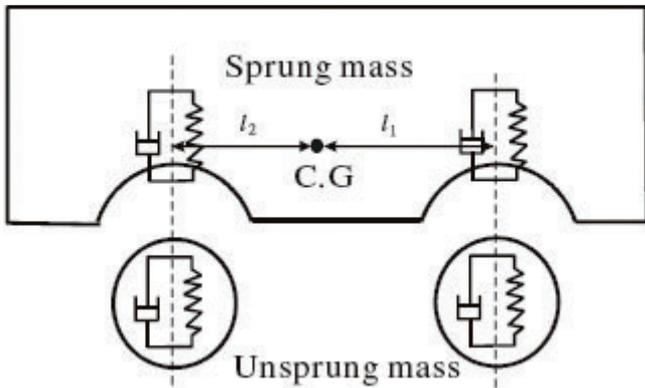


Figure 1: Physical model of half car suspension.

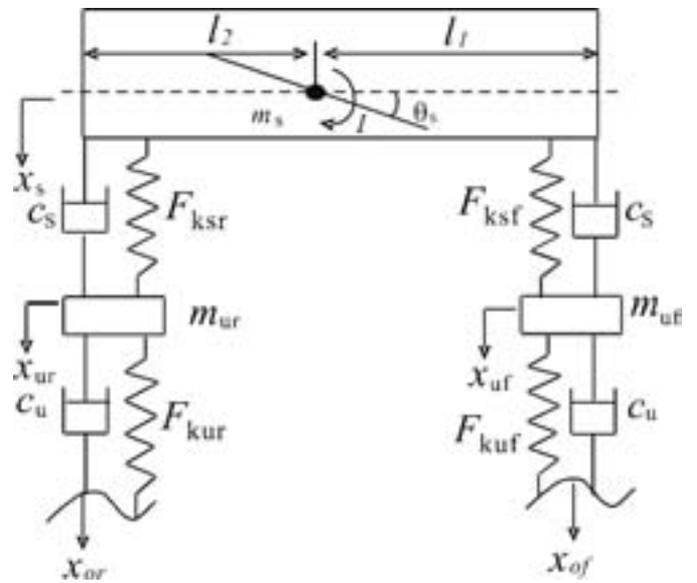


Figure 2: Analytical model of half car suspension.

respectively for vehicle sprung mass, and bounce motion x_{ur} and x_{uf} for rear unsprung mass and front unsprung mass respectively. In this model, the whole sprung mass is considered as m_s and its moment of inertia is I . Front unsprung mass, front wheel with its associated parts, is denoted as m_{uf} and rear unsprung mass, rear wheel with its associated parts, is denoted as m_{ur} . F_{ksr} , F_{ksf} , F_{kur} and F_{kuf} are the forces exerted by nonlinear spring. c_s and c_u are the damping coefficients associated with sprung mass and the unsprung mass respectively. It is assumed that the vehicle during travelling always remains in contact with the ground.

Bounce and Pitch Motion of the Sprung Mass:

The general equation of bounce motion of the sprung mass can be written as from Figure 2:

$$F_{ksr} = k_{sr}(x_s - l_2\theta_s - x_{ur}) + \beta_s \times k_{sr}(x_s - l_2\theta_s - x_{ur})^3 \quad (1)$$

F_{ksr} and F_{ksf} possess the linear and nonlinear force terms for the rear and front suspension of leaf spring respectively and the expressions are calculated as:

$$F_{ksr} = k_{sr}(x_s - l_2\theta_s - x_{ur}) + \beta_s \times k_{sr}(x_s - l_2\theta_s - x_{ur})^3$$

and

$$F_{ksf} = k_{sf}(x_s + l_1\theta_s - x_{uf}) + \beta_s \times k_{sf}(x_s + l_1\theta_s - x_{uf})^3$$

where k_{sr} and k_{sf} are the linear terms of the rear spring and front spring respectively associated with sprung mass and β_s is the nonlinear term of the front and rear spring.

The general equation of pitch motion of the sprung mass can be written as from Figure 2:

$$I\theta_s = F_{ksr}l_2 + c_s(x_s - l_2\theta_s - x_{ur})l_2 - F_{ksf}l_1 - c_s(x_s + l_1\theta_s - x_{uf})l_1 \quad (2)$$

Bounce Motion of the Unsprung Mass:

The general equation of bounce motion of the rear unsprung mass can be written as from Figure 2:

$$m_{ur}\ddot{x}_{ur} = -F_{kur} - c_u(\dot{x}_{ur} - \dot{x}_{or}) + F_{ksr} - c_s(\dot{x}_{ur} + l_2\dot{\theta}_s - \dot{x}_s) \quad (3)$$

$$F_{kur} = k_{ur}(x_{ur} - x_{or}) + \beta_u \times k_{ur}(x_{ur} - x_{or})^3$$

where, k_{ur} and β_u is the linear and nonlinear terms of the spring associated with unsprung mass respectively and $x_{or} = a \sin(\omega t)$ is the rear wheel excitation from the rough surface of the road.

The general equation of bounce motion of the front unsprung mass can be written as from Figure 1:

$$m_{uf}\ddot{x}_{uf} = -F_{kuf} - c_u(\dot{x}_{uf} - \dot{x}_{of}) + F_{ksf} - c_s(\dot{x}_{uf} - l_1\dot{\theta}_s - \dot{x}_s) \quad (4)$$

$$F_{k_{uf}} = k_{uf}(x_{uf} - x_{of}) + \beta_u \times k_{uf}(x_{uf} - x_{of})^3$$

where, k_{uf} and β_u are the linear and nonlinear terms of the spring associated with unsprung mass and $x_{of} = a \sin(\omega t - \tau)$ is the front wheel excitation from the rough surface of the road. a is the amplitude of excitation and time leading in front wheel $\tau = (l_1 + l_2)/V$. Angular velocity of wheel $\omega = 2\pi(V/\lambda)$ and λ is the pitch length.

3. Simulation Parameters

Simulation has been done by MATLAB SIMULINK at steady state condition. Parameters used in this calculation are compatible to the heavy vehicle bus and the fixed parameters are: $m_s = 7000 \text{ kg}$, $I = 30000 \text{ kg-m}^2$, $m_{ur} = 900 \text{ kg}$, $m_{uf} = 600 \text{ kg}$, $k_{sr} = 700,000 \text{ N/m}$, $k_{sf} = 450,000 \text{ N/m}$, $k_{ur} = 4000,000 \text{ N/m}$, $k_{uf} = 3000,000 \text{ N/m}$, $I_1 = 3.2 \text{ m}$, $I_2 = 1.8 \text{ m}$, $\lambda = 0.3 \text{ m}$. The natural frequencies, for the above values without considering the nonlinear terms, are 1.87 Hz for bounce motion of sprung mass, 2.25 Hz for pitch motion and, 11.53 Hz and 12.1 Hz for bounce motion of unsprung masses. It is practical that if the natural frequency of the sprung mass is more, the acceleration to the body of the passenger will be also more and passenger will feel discomfort. For normal passenger car, the recommended natural frequency is below 1.5 Hz. But for heavy vehicle due to the heavy stiffness of spring, up to 3 Hz is allowable. Natural frequency of pitch motion should be near to the natural frequency of bounce motion. Otherwise, during the pitch motion natural frequency, the bounce motion will also be affected. The recommended pitch natural frequency is 1.2% of the bounce natural frequency. But sometimes it is very difficult to keep the pitch frequency within this range. The unsprung mass natural frequency should lie outside the frequency range of vibration to which the human body is most sensitive. At least the bounce natural frequency should not less than 8 Hz, and above 10 Hz gives the good result.

4. Simulation Results

(a) Steady state resonance curve investigation:

A steady state resonance curve is shown in **Figures 3(a)** and **(b)** for the fixed parameters mentioned above and variable parameters are: $c_s = 500 \text{ N-s/m}$, $c_u = 1000 \text{ N-s/m}$, $\beta_u = \beta_u = 0.3$ and $a = 0.01 \text{ m}$. x axis denotes the frequency and the y axis denotes the amplitude in meter for

the bounce motions of sprung and unsprung masses, and radian for the pitch motion of sprung mass. The backbone curve is shown at the natural frequencies and sudden falls of amplitude at the natural frequencies are observed due to the cubic nonlinearity. The maximum amplitude is shown around 0.55 m and 0.2 rad for the sprung mass. In the other frequencies, sprung mass amplitude remains very low.

Now, if the variable parameters are changed to $c_s = 3000 \text{ N-s/m}$, $c_u = 10,000 \text{ N-s/m}$ with the same $a = 0.01 \text{ m}$ shown in **Figure 4**, the amplitude for the sprung mass becomes much lower than the previous one. Further increase of the parameters to $c_s = 15000 \text{ N-s/m}$, $c_u = 30,000 \text{ N-s/m}$ shown in **Figure 5**, the amplitude for the sprung mass shown very low which can be very much comfortable for the passengers on the bus.

(b) Transmissibility investigation

A displacement transmissibility ratio (TR) investigation has been done for this suspension system. TR is defined as $|x_s|/|x_0|$. If the TR is very high then the no contact motion of the wheel with the road surface can be observed and that could be very harmful for the driver to handle the bus. **Figures 6** and **7** show that if the damping coefficient is increased, the TR can be decreased to a very low ratio which can give the good handling to the driver to drive.

(c) rms acceleration investigation:

Acceleration to the passenger can hugely affect to the comfort of the passengers. Ride comfort level (RCL) can be determined from the acceleration rms value [7]. The equation to measure the RCL is:

$$RCL = 20 \log_{10} \left(\frac{A_{rms}}{A_{ref}} \right) \text{ db}$$

Where A_{rms} is the rms acceleration and A_{ref} is the reference value which is taken in Korea 10^{-6} m/s^2 . Now ride comfort level is shown in **Table 1**:

TABLE 1. Ride comfort level scale in Korea.

RCL in Korea	Ride Comfort
103	Very comfortable
103-108	Comfortable
108-113	Medium
113-118	Uncomfortable
118	Very uncomfortable

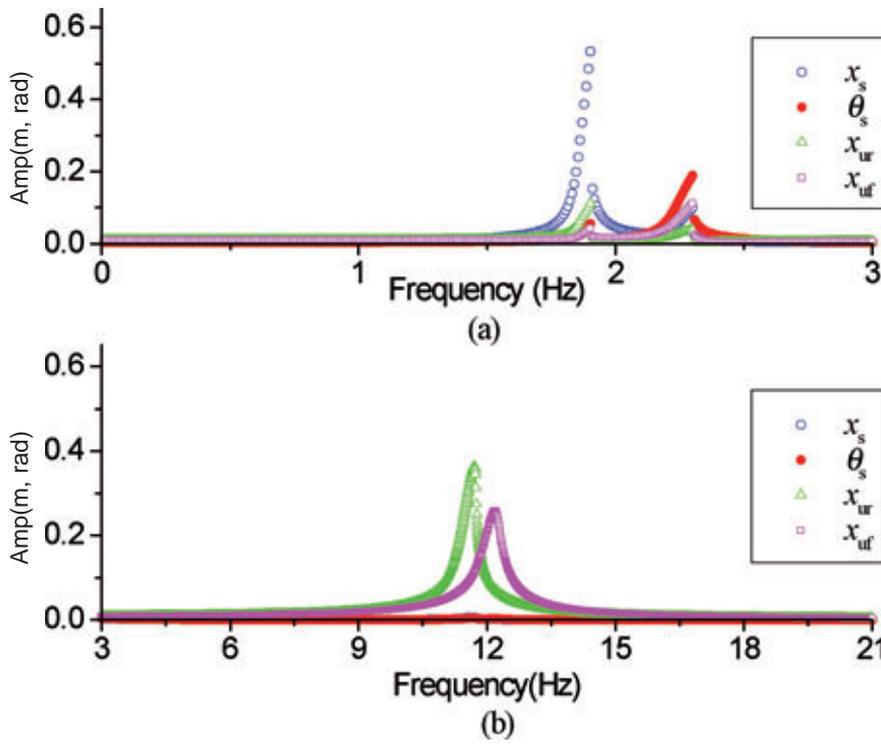


Figure 3: Resonance curve of sprung and unsprung masses for $c_s=500$ N-s/m, $c_u=1000$ N-s/m, $\beta_s=\beta_u=0.3$ and $a=0.01m$.

Figure 4: Resonance curve of sprung and unsprung masses for $c_s=3000$ N-s/m, $c_u=10000$ N-s/m, $\beta_s=\beta_u=0.3$ and $a=0.01m$.

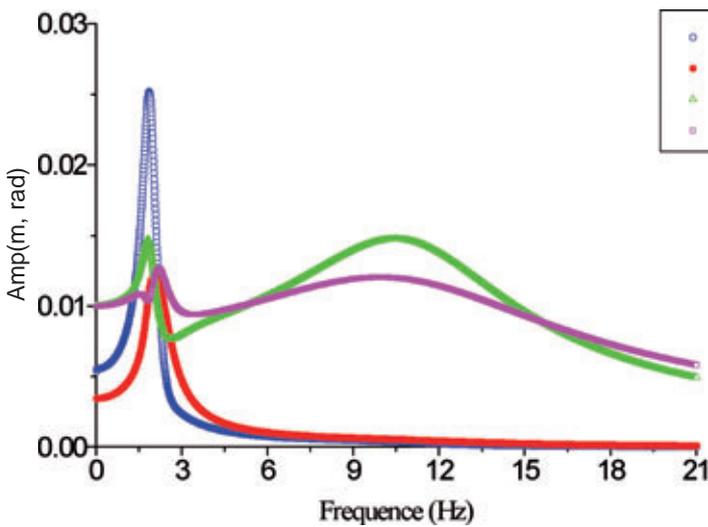
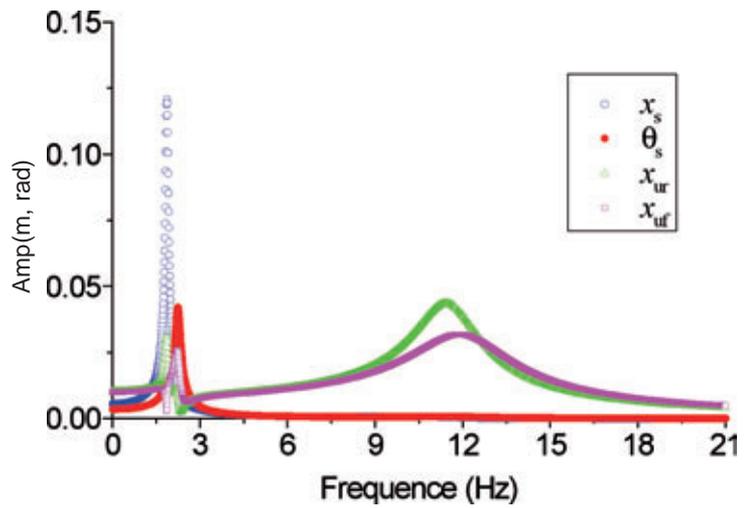


Figure 5: Resonance curve of sprung and unsprung masses for $c_s=15000$ N-s/m, $c_u=30000$ N-s/m, $\beta_s=\beta_u=0.3$ and $a=0.01m$.

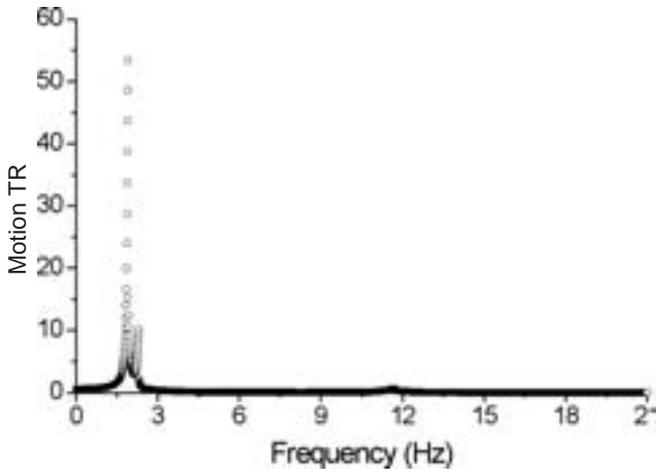


Figure 6: Bounce Motion Transmissibility Ratio of sprung mass for $c_s=500$ N-s/m, $c_u=1000$ N-s/m, $\beta_s=\beta_u=0.3$ and $a=0.01$ m.

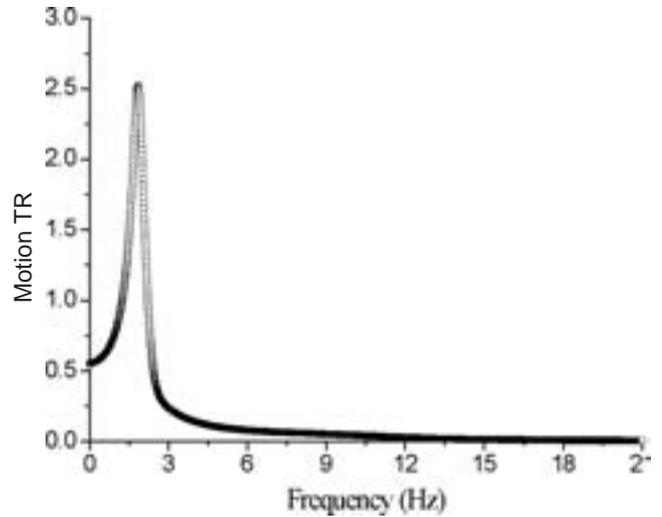


Figure 7: Bounce Motion Transmissibility Ratio of sprung mass for $c_s=15000$ N-s/m, $c_u=30000$ N-s/m, $\beta_s=\beta_u=0.3$ and $a=0.01$ m.

RCL is investigated in **Figure 8** for $c_s=15000$ N-s/m, $c_u=30,000$ N-s/m with amplitude $a=0.01$ m. It is observed that many data show the discomfort level. It is observed that if the damping coefficients of sprung and the unsprung mass decreases, the values of RCL become more which represents the more discomfort to the passengers. Again, if amplitude is changed to $a=0.005$ m with the same other parameters of **Figure 8**, **Figure 9** shows that only some of the frequency zone shows discomfort to the passengers. The ride comfort can be achieved by changing the damping coefficients, more smooth road surface and

also the lowering the natural frequencies of the sprung masses.

(d) Comparison to linear spring model:

It is observed that when the sprung mass vibration is high at the resonance, the peak amplitude of the sprung mass is lower in the nonlinear model than linear model shown in **Figure 10** compare with the same parameters of **Figure 3**. The occurrence of lowering the amplitude in nonlinear spring due to the spring hardening if the vibration is high.

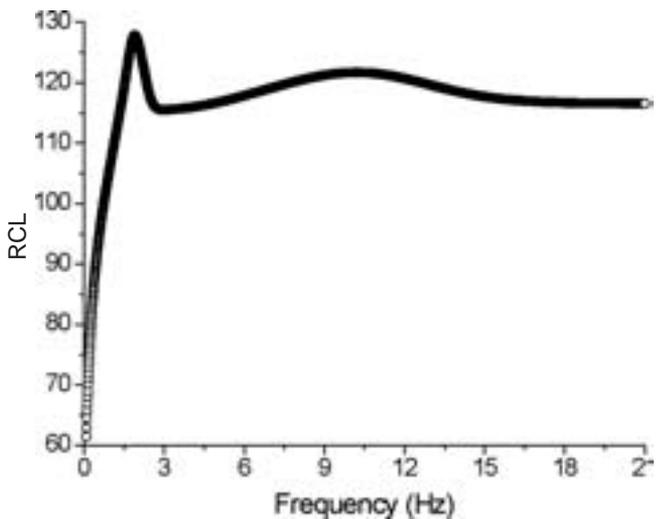


Figure 8: Ride Comfort Level of sprung mass (bounce motion) for $c_s=15000$ N-s/m, $c_u=30000$ N-s/m, $\beta_s=\beta_u=0.3$ and $a=0.01$ m.

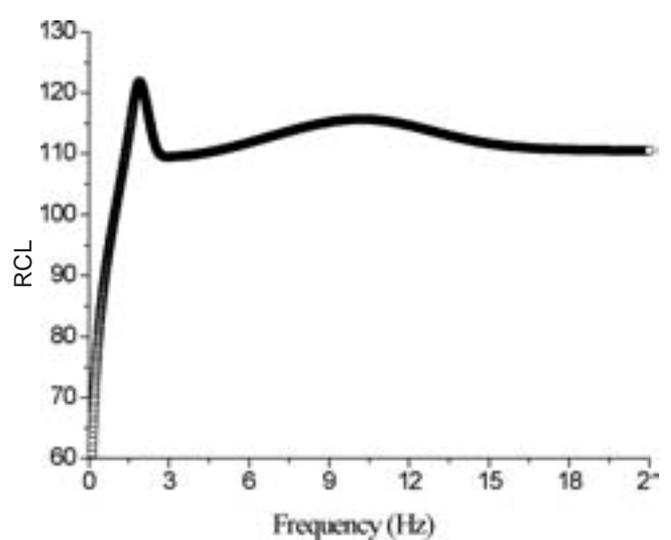


Figure 9: Ride Comfort Level of sprung mass (bounce motion) for $c_s=15000$ N-s/m, $c_u=30000$ N-s/m, $\beta_s=\beta_u=0.3$ and $a=0.05$ m.

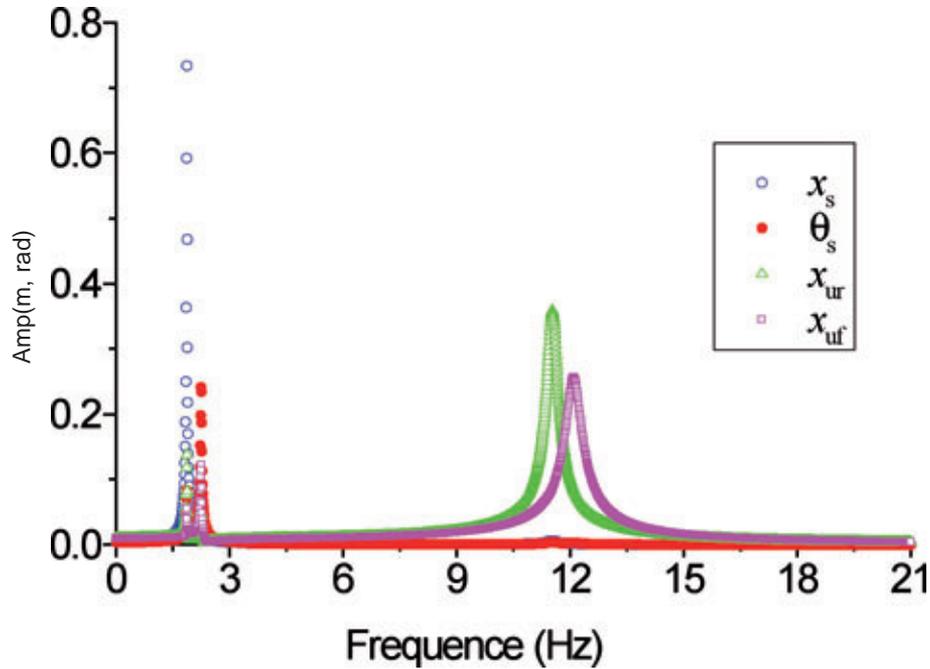


Figure 10: Resonance Curve of sprung and unsprung masses for $c_s=500$ N-s/m, $c_u=1000$ N-s/m, $\beta_s=\beta_u=0.0$ and $a=0.01$ m.

Conclusion

A nonlinear spring model with 4 DOF system of the heavy vehicle car suspension has been investigated by MATLAB SIMULINK. The observations from the investigations are as:

- (i) The nonlinear model shows the backbone curve at the natural frequencies and sudden falls of amplitude to a very low value when the amplitude is high.
- (ii) The nonlinear spring model compare to the linear spring model can show lower peak amplitude.
- (iii) Steady state amplitude of vibration can be reduced significantly by choosing the proper dampers.
- (iv) Providing Comfort to the passengers can be achieved by choosing proper damper, road surface and natural frequencies of the sprung mass.

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