1. Introduction

Over the past decade, various properties of dense electron-hole plasma generated in a semiconductor under high-power ultra short light pulse received notable attention of experimentalists. Among those properties is the superluminescence of short-lived plasma generated in a GaAs sample by a very intense 10 ps light pump (Ageeva et al. 2005). When the total (over time) energy of stimulated radiation escaping from the butt of the sample is plotted against the distance between the active region and the end, it reveals a sort of spatial periodicity. The similar periodicity is shown to exist when the total energy of the stimulated radiation is measured against the photon energy and the delay time between two consecutive pumping pulses. It was suggested that the found periodicity could be due to some modulation of the output radiation with a period of about 4 ps. However, any coherent theoretical explanation of this phenomenon has not been yet proposed.

In the present paper, we show that the amplitude modulation of superluminescence in semiconductor with a characteristic time scale of order 1 ps could be caused by the relaxation oscillation of local disturbances of the quasi-Fermi energy distribution of nonequilibrium electrons.

A high concentration of electron-hole pairs is generated in a direct gap semiconductor whenever the inter-band absorption of short pulses of high-intensity light takes place. Electron-electron collisions establish a quasi-Fermi distribution of nonequilibrium electrons and holes over a quite short time of about $10^{-14}$ s. Further behavior of electron-hole plasma during the action of the light pump pulse is determined by the kinetics of its cooling and by the recombination of electrons and holes. If the conditions of population inversion is maintained in the electron-hole plasma, then the amplification of spontaneous radiation in the form of superluminescence will be possible (Pankove, 1971).

Earlier, it had been suggested that the picosecond stimulated radiation and its modulation were determined by the deviation of the distribution of carriers from the quasi-Fermi distribution (Ageeva et al., 2007; Altybaev et al., 2004; Altybaev et al., 2009). According to Altybaev et al. (2004), the disturbance of the quasi-Fermi distribution of nonequilibrium electrons takes place at certain energies of electrons, $e$ and $e + \hbar\omega_0$ (where $\hbar\omega_0$ is the energy of optical phonon). The disturbance of the quasi-Fermi distribution of nonequilibrium electrons close to the bottom of the conduction band is due to the radiative transitions of electrons with energy $e$ to the valence band.
as well as to the emission of optical phonon with energy \( e + \hbar w_0 \). The disturbance of the electron distribution at energy \( e + \hbar w_0 \) occurs due to the emission of optical phonon by electron with energy \( e + \hbar w_0 \) and the absorption of optical phonon by electron with energy \( e \). As to the distribution of nonequilibrium holes, it remains almost similar to the Fermi type because of the huge difference in density of states between the conduction and valence bands bearing in mind the momentum and energy conservation in hole-phonon interactions.

**2. The Model**

Our approach takes advantage of the rate equations formalism (Carroll, 1985). Let \( N \) and \( S \) be the respective densities of electrons and photons.

The dynamics of \( N \) is governed by the following processes:

(i) **Pumping.** Each second the optical pump creates \( P \) carriers in the unit volume of the active region and this inflow of carriers can be considered constant while the light pulse lasts;

(ii) **Stimulated radiation.** Due to the stimulated radiative transitions, the density of electrons tends to relax to the threshold of transparency \( N_{th} \) (corresponding to the onset of population inversion) with the rate \( gS \), where \( g = ca/\mu \) is the differential gain, \( c \) is the speed of light in vacuum, \( a \) is the gain constant, \( \mu \) is the group refractive index, and \( S \) is the density of photons;

(iii) **Spontaneous recombination of carriers.** Electrons and holes recombine with the time constant \( \tau_{sp} \). This process results in spontaneous radiation;

(iv) **“Healing” of a disturbance in the quasi-Fermi distribution.** The quasi-Fermi distribution of electrons is characterized by the density of electrons \( N_e \). Electron-electron collisions tend to “heal” any deviation from the quasi-Fermi distribution with the time constant \( \tau_h \) (Gantmacher and Levinson, 1987).

The dynamics of \( S \) is determined by a generation of photons owing to the stimulated radiation and by their loss due to the propagation of light in the medium and intraband optical absorption with the effective time constant \( \tau_s \).

Taking into consideration the abovementioned mechanisms leads to the following coupled rate equations for the densities of photons and electrons:

\[
\dot{S} = g(N - N_{th})S - \frac{S}{\tau_s},
\]

\[
\dot{N} = P - g(N - N_{th})S - \frac{N}{\tau_{sp}} - \frac{N - N_{th}}{\tau_h}. \tag{1}
\]

(The dot denotes \( d/dt \).)

The typical values of the different parameters in the model (1) are given in Table 1. Hereinafter we assume \( \tau_{sp} \gg \tau_h \).

**TABLE 1. The parameters of the model.**

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>2.5x10^{-16} cm^2</td>
<td>Agrawal and Dutta (1993)</td>
</tr>
<tr>
<td>( m )</td>
<td>5.1</td>
<td>Ageeva et al. (2005)</td>
</tr>
<tr>
<td>( N_{th} )</td>
<td>10^{18} cm^{-3}</td>
<td>Agrawal and Dutta (1993)</td>
</tr>
<tr>
<td>( t_s )</td>
<td>10^{-11} s</td>
<td>Agrawal and Dutta (1993)</td>
</tr>
<tr>
<td>( t_{sp} )</td>
<td>10^9 s</td>
<td>Agrawal and Dutta (1993)</td>
</tr>
<tr>
<td>( t_h )</td>
<td>10^{-12} - 10^{-11} s</td>
<td>Gantmacher and Levinson (1987), estimate</td>
</tr>
</tbody>
</table>

Eqs. (1) can be converted into dimensionless form by performing the linear scaling

\[
S = \frac{s}{\tau_{th}}, \quad N = \frac{n + 1}{\tau_{th}} + N_{th}, \quad P = \frac{p + 1}{\tau_{th}} \frac{N_{th} - N_{th}}{\tau_{th}}, \tag{2}
\]

and they become

\[
\begin{aligned}
\dot{s} &= \frac{ns}{\tau_s}, \\
\dot{n} &= \frac{p - (n + 1)s - n}{\tau_h}.
\end{aligned} \tag{3}
\]

Note that now the dimensionless \( n \) and \( p \) are no longer proportional to their dimensional prototypes, but rather are deviations of the corresponding absolute quantities from the threshold of the stimulated radiation.
3. Results and Discussion

System (3) has two steady-state solutions:

\[ \bar{x}_1 = 0, \quad \bar{p}_1 = p; \]

\[ \bar{x}_2 = p, \quad \bar{p}_2 = 0. \]

Steady state (4) is always a saddle, since eigenvalues of the Jacobian matrix \( J \) of Eq. (3), being the roots of the characteristic polynomial

\[ \det(\lambda I - J) = \lambda^2 - \lambda \text{tr} J - \det J, \]

have opposite signs: \( \lambda_1 = -\tau_h^{-1}, \lambda_2 = p / \tau_s. \)

Steady state (5) is always a stable focus/node, because

\[ \lambda_2 = -p - 1 \pm \sqrt{(p + 1)^2 - 4 p \tau_h / \tau_s}. \]

States (4) and (5) switch their stability via transcritical bifurcation when the parameter \( p \) passes through 0.

For steady state (5) to be a focus (corresponding to the damped oscillations), the discriminant of the quadratic characteristic equation has to be negative:

\[ \Delta = (\text{tr} J)^2 - 4 \det J = (p^2 - 2 p (2 \tau_h / \tau_s - 1) + 1) / \tau_h^2 < 0 \]

This inequality is equivalent to the conditions

\[ \frac{2 - \sqrt{1 - \varepsilon}}{\varepsilon} < p < \frac{2 + \sqrt{1 - \varepsilon}}{\varepsilon}, \]

where

\[ \varepsilon = \frac{\tau_s}{\tau_h}. \]

Thus, the existence of oscillations requires the “healing” rate to be slower than the photon loss rate. It also imposes restrictions on the intensity of pump. The allowed range of \( p \) is wide when \( \varepsilon \ll 1 \), \( 0 < p < 4 / \varepsilon \), and it shrinks to \( 1 - 2 \sqrt{1 - \varepsilon} < p < 1 + 2 \sqrt{1 - \varepsilon} \) as \( \varepsilon \) gets closer to 1.

The angular frequency of oscillations is given by formula

\[ \omega = 3 \lambda = \frac{\sqrt{4 p \tau_h / \tau_s - (p + 1)^2}}{2 \tau_h}. \]

Estimating the frequency and the period from Eq. (8) involves knowledge of pump rate used in the experiment, but it is lacking. Instead, we have to resort to a kind of a trick.

According to Eq. (5), in steady state dimensionless pump \( p \) simply equals dimensionless density of photons \( \bar{p} \). The latter can be converted to the dimensional density \( S \) using Eq. (2). This yields for the frequency

\[ \omega = \sqrt{g S / \tau_s - \left( \frac{g S \tau_h + 1}{2 \tau_h} \right)^2} \]

In the extreme case of \( \varepsilon^2 \ll g S \tau_s \ll 1 \), Eq. (9) reduces the formula obtained earlier by Lau and Yariv (1985) for lasers:

\[ \omega = \sqrt{g S / \tau_s} \]

The steady-state density of photons is related to measurable quantity - the intensity of light, \( I \), escaping from either end of the sample - by equation

\[ I = \frac{1}{2} (h \nu / \mu) S \]

In work of Ageeva et al. (2005), the flux of the stimulated radiation is reported to be at least 100 MW/cm² with the energy of individual photons 1.4 eV. Interpreting that as a steady-state flux, we get for \( S \) the estimate \( 1.5 \times 10^{17} \) cm⁻³. Putting this value in Eq. (9) yields \( \omega = 1.36 \pm 1 \times 10^{12} \) s⁻¹ for \( \tau_s = 1 \) ps, that corresponds to a period of 4.6 ps.

Figure 1 illustrates that at large, the frequency of the damped oscillations weakly depends on the characteristic time of “healing” (except in the vicinity of the very onset of the stable focus) and quickly comes to a plateau where it is solely controlled by the photon loss rate.

\[ \text{Figure 1. Frequency of relaxation oscillations as a function of the characteristic time of “healing”.} \]
Conclusion

Experimentally observed amplitude modulation of superluminescence stimulated in a semiconductor by an ultra-short high-power light pulse is likely to be explained by the relaxation oscillations. Those damped oscillations with the natural period of a few picoseconds are the manifestation of intrinsic resonance in which energy stored in the nonlinear system oscillates back and forth between the carrier and photon populations. The remarkable feature of the oscillations is that their frequency is essentially controlled by the photon loss rate and rather weakly depends on the rate with which the disturbances in the quasi-Fermi distribution of nonequilibrium electron-hole plasma fade out.

References